

國立中央大學數學系

專題演講

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演講題目：**Properties of One and Two Dimensional Random Walks: Simple and Conditioned**

演講時間：2023年06月30日(星期五) 3:00 p.m. ~ 4:00 p.m.

演講地點：中央大學鴻經館M107

Abstract：

We consider random walks on $\mathbb{Z}^d$, $d = 1, 2$ in case simple and conditioned on never hit the origin. The simple random walk starting at a point $x \in \mathbb{Z}^d$ is defined as

$$S_n = x + X_1 + X_2 + \cdots + X_n$$

whereas the conditioned one is

$$\hat{S}_n = x + \hat{X}_1 + \hat{X}_2 + \cdots + \hat{X}_n.$$

They are both Markov chains with transition probabilities

$$\mathbb{P}[S_n = y | S_{n-1} = x] = \frac{1}{2d} \quad \text{if } \|y - x\| = 1,$$

and

$$\mathbb{P}[\hat{S}_n = y | \hat{S}_{n-1} = x] = \begin{cases} \frac{1}{2d} \frac{a(x+y)}{a(x)} & \text{if } x \neq 0 \text{ and } \|y - x\| = 1 \\ 0 & \text{otherwise} \end{cases}$$

here $a(x)$ is the potential kernel of S_n . Let τ and $\hat{\tau}$ be the exiting time of a connected finite subset of $\mathbb{Z}^d$ with respect to S and $\hat{S}$. τ and $\hat{\tau}$ are finite almost surely. We will give an expression of their distribution and expectation in terms of transition matrix restricted on D and the Green function on D . The simple random walk are martingale but the conditioned is a strictly submartingale. We also give necessary and sufficiency condition such that a function of $\hat{S}_n$ and n is a martingale.

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